

Developing interactive applets with GeoGebra: processes, technologies

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Abstract

With continuing technological improvements, it is increasingly easy to produce high-quality interactive online resources to support learning of mathematics & statistics concepts. Software like GeoGebra (www.geogebra.org) allows users to produce dynamic, interactive constructions, with relatively low technical demands and moderate time and resource requirements. In this paper, we describe a project at the School of Mathematics & Statistics, The University of Melbourne, to produce a collection of interactive applets to enhance teaching in a range of undergraduate subjects offered by the school. The applets target specific learning & teaching needs of our school and are tailored precisely to our local learning & teaching context. We will give an overview of the project, and describe the approach taken by the project to identify teaching needs that could be serviced by new applets, and the design and feedback processes used to produce the applets. We give a preliminary survey of the outcomes of the project and discuss its impact on teaching & learning, both directly on the student experience as well as on academic staff through professional development. We offer a discussion of the technical merits and drawbacks of GeoGebra as a technological platform for the development of interactive applets for undergraduate mathematics and statistics, including data on the development time required to produce applets using GeoGebra.

1 Introduction

1.1 Applets in mathematics and statistics education

Applets, small pieces of software which serve a specific task and run within a web browser, are used widely in undergraduate mathematics and statistics. Applets can offer interactive, dynamic visual representations of mathematical & statistical concepts. The interactivity that applets offer can "extend and enhance" the communicative power of graphical representations of mathematical concepts (1). Applets often have a specific conceptual focus (for example (2, 3)), so can be used selectively by instructors to support understanding of key concepts or to enhance an instructor's 'story-telling' or educational narrative. They are flexible, allowing use in a classroom or by students outside of class, and are usually easy for users to master without training or previous experience with their use. Although many of these benefits are not unique to applets, the potential flexibility and ease of use that applets offer make them a popular choice of computer-based learning resource. Research shows that computer-based tools, such as applets, can be effective in mathematics and statistics education (4). Many collections of applets exist on the web; for instance MERLOT (www.merlot.org) or GeoGebraTube (www.geogebraTube.org), although the resources they provide can be of varying quality.

Applets may be built with a range of technological platforms. Java (www.java.com) was the most common platform for applets through the late 1990's and 2000's, and is still used widely; however, improvements in web browser technology mean that applets can now be built using HTML5, which run in a web browser without requiring any additional browser plugins or other software. An example of an applet is shown in Figure 1.

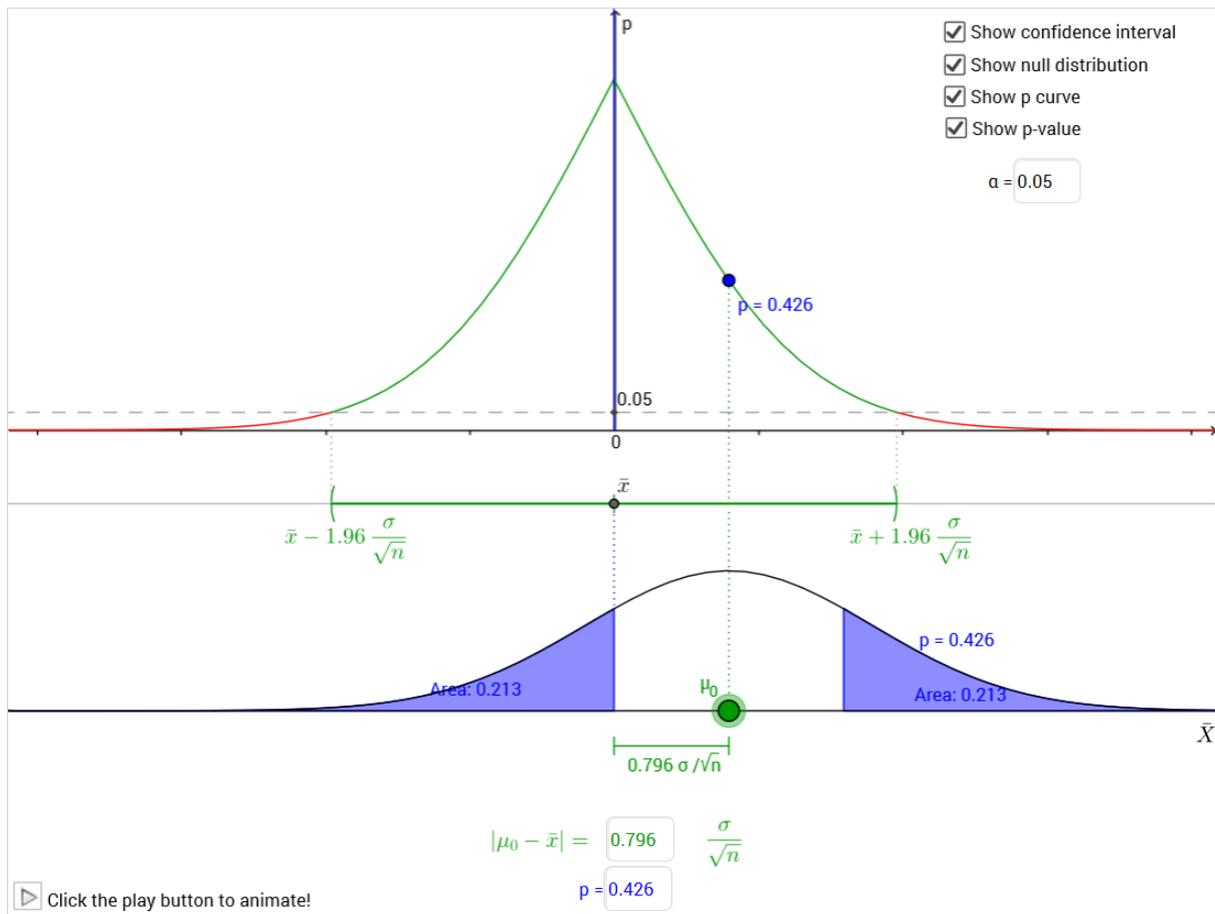


Figure 1: Screenshot showing an applet linking confidence intervals, hypothesis testing and p-values. Surrounding web browser window not shown. This applet is accessible at <http://www.melbapplets.ms.unimelb.edu.au/?portfolio=confidence-intervals-hypothesis-testing-and-p-values>

1.2 GeoGebra

GeoGebra (www.geogebra.org) is a dynamic geometry, algebra and calculus software system (5). It is open-source software and freely available to download. Development of GeoGebra and related research is led by a non-profit foundation, the International Geogebra Institute (6). Materials produced in GeoGebra can be uploaded to the web via the GeoGebraTube repository (www.geogebraTube.org) or embedded within another website such as a learning management system (LMS). An outline of the technical aspects of GeoGebra, and a discussion of the strengths and weaknesses of GeoGebra as a platform for applet development, is given in section 3.

2 Project: *Conceptual learning with interactive applets*

2.1 Genesis of the project

Applets have been used in the School of Mathematics & Statistics at the University of Melbourne, in various ways and in several subjects, for many years. However, despite a wealth of applets publicly available on the web, many of the applets being used were not perfectly suited to the local teaching context. For instance, the notation or terminology used in an applet may differ from that used in lecture notes, or between one applet and the next; some applets contained superfluous features, or lacked features which would have been of use; others have technological obstacles which make their use difficult. Despite the efforts of projects like MERLOT (7) to provide quality control for online teaching resources through peer review, it is often still difficult to find resources that meet a particular combination of teaching needs. During discussions between the authors, it was realised

that, due to technological advancement over recent years, it is now feasible for academics to build custom-made interactive applets, tailored precisely to their needs, with relatively moderate requirements of time or technical expertise. As a demonstration of this, the authors produced two applets using GeoGebra, a software package which one of the authors had recently become acquainted with. The applets were designed around demonstrations used in undergraduate statistics lab classes, and anecdotally showed good potential for enhancing conceptual understanding in early trials with students and colleagues. The potential for such resources to make a notable impact on teaching & learning in the department for only a moderate investment of resources was recognised.

A small amount of seed funding was obtained from the School for further applet development. This funding allowed us to produce two additional statistics applets, with the development done by a graduate student research assistant. A larger grant of \$10,000 was obtained from the University to extend the project to several first and second-year mathematics and statistics subjects. The main aim was to enhance learning by producing new applets targeting specific learning & teaching needs in our school which were not met by existing resources. The applets would be tailored precisely to the local learning & teaching context. An additional aim of the project was to develop a pool of expertise in development and use of such applets, which may be drawn upon in the future by other academics in the school seeking to produce or use similar resources for their own needs. A further aim, although more tacit, was professional development: to trigger colleagues to reflect on their teaching practice, by prompting them to make explicit their pedagogical goals and approaches during the applet design and feedback process.

2.2 Identifying teaching needs

Once the project was underway, areas in the school's core undergraduate programme were identified where new applets could have a positive impact on teaching & learning. This was done in discussion with interested teaching staff as well as drawing on the team members' own experiences. Areas were sought which were inadequately served by existing interactive resources, and for which the project could make a contribution within its resource and technical capabilities. Such areas were identified in a range of subject areas, including introductory and intermediate statistics and probability, first-year calculus, second-year real analysis, and first-year mathematics for biomedical sciences.

The areas identified were driven largely by three main motivators, which aligned with the motivations of the academics participating in the project. Each of the three motivators feeds in to the main aims of the project. This is discussed in section 2.7.

2.3 Processes: Design and feedback

Once an area for an applet had been identified, team members would briefly survey existing resources to identify anything that might be pertinent. The team would produce draft designs for one or more applets, in consultation with the relevant academics when appropriate. These designs would be implemented in GeoGebra either by a research assistant or by a member of the project team (the authors of this paper). The resulting applet(s) would then undergo a lengthy cycle of review, feedback and refinement. The applet(s) were provided to relevant academics who were encouraged to play with them, with particular attention to whether they met their teaching needs. In some cases, academics would be prompted with specific questions, such as what terminology or notation should be used in the applet, what key examples they would want the applet to support, or what additional functionality would be needed to make effective use of the applet in their teaching. Academics would provide feedback and suggestions to the project team, who would revise the applets accordingly. This cycle would be repeated several times. A further cycle would usually be

performed after the applets were used by staff or students in the course of teaching. The duration of this process, from first discussions to finished product, was typically 2-3 months.

The project team also ran workshops with academics to refine and test the applets. Two workshops were held. One focussed on statistics applets and targeted practicing statisticians and staff involved in teaching statistics; the other focussed on mathematics applets and targeted lecturers of core first and second-year mathematics subjects. Each workshop had about 10 participants and ran for about 90 minutes. In the workshops, brief demos of select applets were given, and then participants were invited to play with the applets, individually or collaboratively, and give constructive feedback. These workshops prompted discussion about subtle pedagogical and design issues, brought to light through collaborative discussion, which were not identified in the previous feedback cycles. Further revisions were made to the applets following the workshops.

Several design principles informed the design and development of the applets. These include the use of colour coding to re-inforce semantic relationships between visual elements; the use of directly manipulable user interface elements, acknowledging that knowledge is embodied; and the use of representations, notation and terminology consistent with other course materials, such as lecture notes, to ensure coherence with existing resources. These principles are guided by cognitive load theory (8) and computer user interface research (9), and broadly align with those of similar projects such as (2, 3). These will be discussed further in a future paper.

In addition, the project also produced supporting resources including instructors' notes, sample online tutorials and sample assessment items. These are discussed further in section 2.4.

2.4 Project outcomes to date

The project has produced 29 applets at time of writing¹. These are available on the project website at <http://www.melbapplets.ms.unimelb.edu.au>. The applets address concepts from a range of subjects and levels, including introductory calculus, statistics and probability, analysis, and discrete mathematics. A list of the concepts addressed by the applets is given in Table 1. These applets range in complexity from simple demonstrations with one or two visual components to rich applets with complex animations and multiple representations of key concepts. Two applets were adapted from existing freely available resources from GeogebraTube; the remainder were designed and built by the project. A screenshot of a typical applet is given in Figure 1.

Subject area	Concepts addressed by applets
Calculus (first-year level)	Sequences & series Inverse of a function Parametric curves (2 applets) Continuity ODEs: direction fields and Euler's method Mathematical models: population models without harvesting and with harvesting, springs (3 applets)
Introductory statistics	Discrete and continuous probability distributions (2 applets) Confidence intervals, hypothesis testing and p-values Power of a hypothesis test

¹ This excludes some applets which are minor variations on other applets, and 3 applets which, at time of writing, are incomplete and still under development.

	Partitioning of variability (2 applets) Normal probability plots
Intermediate probability & statistics (second-year level)	QQ plots Distribution of order statistics Maximum likelihood estimators Central Limit Theorem and Law of Large Numbers * Prisoners' Paradox *
Real analysis (second-year level)	Formal ϵ-M definition of convergence of a sequence Formal ϵ-δ definition of convergence and continuity Differentiability Riemann sums and integral
Discrete mathematics	Difference equations (2 applets) Discrete time population genetics model
Linear Algebra	Linear transformations of the plane and eigenvectors

Table 1: Concepts addressed by applets produced by the project, grouped by subject area. One applet was produced for each concept, unless indicated otherwise.

* These applets have not yet been published publicly so links are not yet available.

In addition to the applets, the project also produced resources to support the use of applets in teaching. Instructors' notes were produced for several applets. The notes begin with a short description of the rationale for the applet, including key concepts for which it was designed to illustrate. This is followed by an outline of the user interface components of the applet. The notes typically also offer suggestions for ways that the applet may be used in teaching, and/or list specific examples (for instance, functions or combinations of parameter values) that can be used with the applet to highlight key aspects of the concept(s) in question. The examples provided have been tested in the applet to ensure that they work smoothly and display clearly. For examples which require a function or equation to be input into the applet, the equation is provided in both standard mathematical notation and in GeoGebra's input notation, so that the input may easily be copied-and-pasted directly into the applet. These documents also note any technical or mathematical details which are relevant to the teaching of the concept but may not be outwardly obvious. Such details may include, for example, any assumptions being made in the mathematical model underlying the applet. The project team felt it important to document such details to enable instructors, when discussing or demonstrating the applet to students, to present a mathematically correct narrative that avoids conveying subtle misconceptions. Overall, the instructors' notes are intended assist teaching staff by reducing technical or cognitive barriers to effective use of the applet.

The project also produced sample online tutorials associated with two of the applets. These take the form of a web page with a sequence of questions guiding students through a learning activity using an applet. These were produced for two statistics applets; statistics was targeted for the tutorials because the statistics subjects offered by our school include computer lab classes in which the online tutorial exercises could potentially be embedded. Further improvement of the tutorials, informed by cognitive learning theory as in (10) and cognitive theory of multimedia learning (11), also remains for the future. The project team is in the process of producing sample assessment tasks, including assignment and online quiz questions, which make use of the applets. These will be published to the project website after they have been trialed and refined.

It was not our aim to produce supporting resources for every applet in the collection; rather, it was intended to produce a few exemplar resources as part of a showcase, which may act as models for further resources produced in the future.

2.5 Integration and evaluation

The design and development phase of the project, during which new applets are produced, is now nearing completion, with most applets complete or approaching the end of the feedback cycle. However, work is continuing to integrate the applets and other resources into subjects offered in the school. Some applets will be used in teaching for the first time during semester 2, 2015. During the next phase of the project we will explore how to further integrate applets into the existing teaching models used in the school, which include tutorials, computer labs, individual consultations, and various forms of assessment, as well as lectures. In some cases it may be desirable to adjust the teaching model to take full benefit of the learning opportunities that the applets offer.

The project also includes an evaluation phase. During the evaluation phase, data will be collected to investigate the project's impact on student learning, as well as on teaching staff in the school. Preliminary evaluation has already been conducted in 4 subjects which made use of the project's applets. This was done using data from online student surveys, as well as from analytics data and from assessment, and results have been positive. We briefly report here on the evaluation results from one subject. In semester 2, 2014, a collection of applets were used in a mathematics subject for first-year biomedicine students. The applets related to difference equations^{2,3}, discrete-time models of population genetics⁴, and first-order ODEs⁵. The applets were used in lectures, and provided for students to access outside of class. Students were required to use one or more of the applets in assignments during the semester. Enrolment in the subject was 183 students, mostly first-year, all enrolled in a Bachelor of Biomedicine. An online survey was administered to the students over 5 days during weeks 7-8 of semester, after the applets had been used in lectures and assignments. The survey contained 6 questions, of which we will discuss only two. These are shown, along with results, in Table 2 below. 38 students responded to the survey, a return rate of 20.8%.

The first question asked students to rate their agreement with the statement "*I found that the applets improved my understanding of concepts from this subject*". The response options were *Strongly agree, Agree, Unable to judge, Disagree, Strongly disagree*. Of the 38 respondents, 34 agreed or strongly agreed with the statement; the remaining 4 chose *Unable to judge*. No respondents disagreed or strongly disagreed with the statement. This question was intended to assess students' perceptions of the impact of the applets on their learning. This strong response indicates that students do indeed believe the applets to be beneficial for their learning.

The fourth question asked "*How did you use applets in this subject? Please select all that apply.*" The response options, and percentage of respondents that selected that option, are shown in Table 2 below. 37% of respondents answered that they used applets to help understand lecture content; 68% used applets to help answer exercises, and 97% (all but one) used applets as part of an assignment. These responses appear to show a preference towards task-oriented use of applets: students favour applet use to help complete specific learning tasks, rather than to assist with a general understanding of concepts. However, in this subject some assignment questions and non-assessed practice exercises explicitly direct students to use applets, which likely contributed to the high rates for the task-oriented responses. Whether the apparent preference for task-oriented use is genuine or a consequence of the explicit directions given in assignments and exercises may be

² <http://www.melbapplets.ms.unimelb.edu.au/?portfolio=iterating-a-difference-equation>

³ <http://www.melbapplets.ms.unimelb.edu.au/?portfolio=cobwebbing>

⁴ <http://www.melbapplets.ms.unimelb.edu.au/?portfolio=population-genetics-by-fhw-model>

⁵ <http://www.melbapplets.ms.unimelb.edu.au/?portfolio=exploring-an-ode>

answered by further evaluation. The remaining questions on the survey addressed technical aspects such as ease-of-use or were open-response. We will not discuss them here.

Q1:	I found that the applets improved my understanding of concepts from this subject.				
	Strongly agree	Agree	Unable to judge	Disagree	Strongly disagree
Number	13	21	4	0	0
%	34%	55%	11%	0%	0%
Q4:	How did you use applets in this subject? Please select all that apply.				
	Number	%	Response		
	14	37%	<i>I used applets to help me understand content from lectures</i>		
	26	68%	<i>I used applets to help me answer exercise sheet questions</i>		
	37	97%	<i>I used applets as part of an assignment</i>		
	17	45%	<i>When I used applets, I was working/studying on my own</i>		
	5	13%	<i>When I used applets, I was working/studying with friends</i>		
	0	0%	<i>I did not use applets in this subject</i>		
	0	0%	<i>Other</i>		
Note: Respondents could choose multiple answers in question 4.					Total responses: 38

Table 2: Select survey questions for first-year biomedical mathematics class and summary of results.

These results indicate that students perceive the applets as beneficial for their learning. Preliminary evaluation results from other subjects (not discussed here) paint a similar picture. More work is required before reaching definite conclusions however, in particular regarding student motivations for applet use. The evaluation will also include interviews with academic staff, which will explore the impact of the project on teaching practice in the school, including its role in professional development.

2.6 Early impacts

So far, the applets produced by the project have been used in 8 subjects over 3 semesters, potentially reaching an estimated 5000 enrolments and over 3000 distinct students. The authors have used the applets in lectures, tutorials, computer labs and one-on-one consultations and have observed first-hand the power of the applets to strengthen understanding of concepts, clear up misconceptions, and support students to see relationships between concepts and to reflect on their learning. An evaluation of the impact on student experience is ongoing. The impact of the project has also been felt more widely in the school. Over 20 academic staff were involved in the development and feedback processes during the project, including teaching specialist staff, research/teaching academics, consulting statisticians and casual tutors. By involving many academic staff from the school in the development and feedback processes, the project helped foster discussion and collaboration between staff, some of whom would not otherwise have been involved in the development of teaching resources. By actively seeking input and feedback from academics, the project prompted academics to reflect on their teaching practice. In this way, the project helped encourage collegiality and reflective practice in a non-confrontational and respectful way. We have only anecdotal evidence for these impacts so far, but this will be the focus of further evaluation in the future.

2.7 A framework for analysing applet impact

During the first phase of the project, the project team met with academics to identify areas in their undergraduate teaching where new applets might be of benefit. The involvement of individual academics in the project, and the development of ensuing applets, were driven largely by three (overlapping) motivators:

1. Pedagogical impetus. The original motivation for the project was to develop powerful interactive visuals for mathematical & statistical concepts. The visuals are tailored to a specific pedagogy (while providing enough flexibility to adapt to other pedagogies if needed), with the aim of strengthening links between internal visual images and mathematical formalism.
2. Reduce technological obstacles. Some lecturers had used existing resources (typically Java-based applets or Excel spreadsheets) in their teaching for many years. These resources generally served the pedagogical requirements adequately, but had in recent years become increasingly difficult to use due to changing software configurations – in particular, stricter browser security settings, which require users to click through several security warnings or modify browser configuration before the software would load. In these cases, the lecturers wanted similar applets that did not require Java or other additional software.
3. Enhance subject delivery. In some subjects, opportunities were identified where interactive applets could enhance existing teaching practice, for instance by replacing a series of hand-drawn diagrams used by lecturers with a single applet. Lecturers typically wanted to enhance engagement and interactivity in lectures using media beyond the conventional lecture slides or document camera.

Two main aims of the project were, ultimately, to improve student learning, and to provide professional development by prompting reflection amongst teaching academics. We posit that each of the three motivators above feeds into these main aims. This is represented schematically in Figure 2. For example, constructing pedagogically tailored visual aids helps students link symbolic and visual representations of concepts, feeding in to improved conceptual understanding. Designing such applets requires teachers to reflect deeply about the concepts in question and the narratives they use to explain them, a form of (informal) professional development. Removing technological obstacles reduces barriers to student engagement, which feeds into improved learning. Designing replacements for old applets necessitates an examination of the role that the applets were playing, leading to reflection on teaching practice. Enhancing subject delivery leads to improved engagement and learning; seeking feedback on a new interactive resource prompts staff to experiment with new examples or narrative sequences. Investigating these links will be a task for the ongoing evaluation phase of the project.

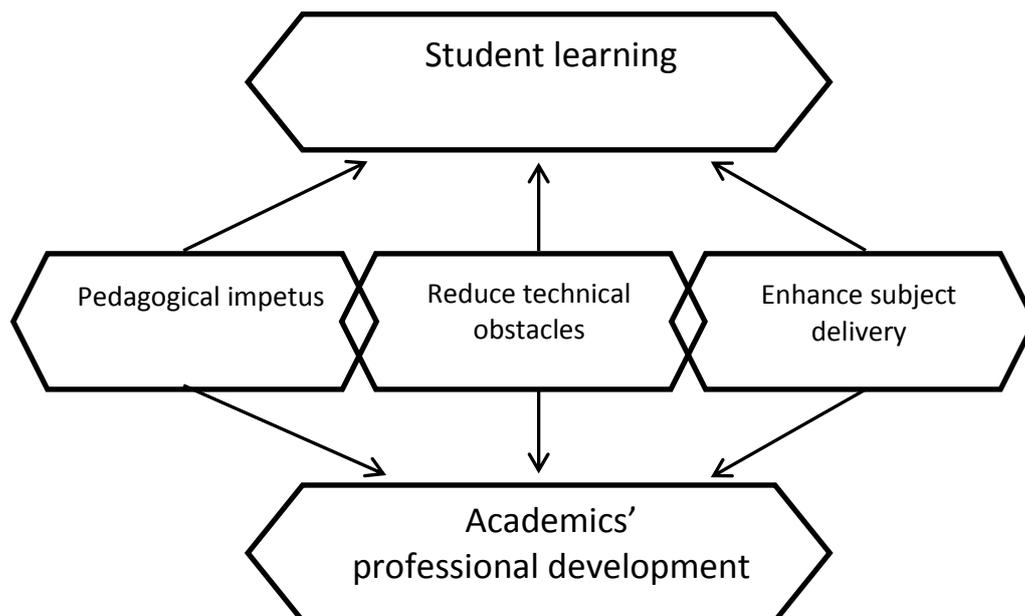


Figure 2: Emerging framework for evaluation of project impact

3 Technology: GeoGebra as a platform for interactive resource development

In this section we will discuss GeoGebra as a technological platform for applet development. We will outline some technical aspects which informed the choice of GeoGebra as a technological platform for our applet development, and discuss some strengths and weaknesses of GeoGebra that we have encountered for this kind of project. We give some data on the development time required to produce applets in GeoGebra, which may be of use to those planning similar projects or for comparison of the relative ease of development of different technological platforms.

GeoGebra is an open-source mathematical software package, freely available at www.geogebra.org. It includes dynamic geometry, calculus, symbolic algebra and statistics functionality. GeoGebra can be used in several ways, including as a standalone desktop application, as a tablet app, as a browser-based web app, or embedded as an applet into a webpage. The GeoGebra project also provide a repository, GeoGebraTube, to which users can upload and share GeoGebra worksheets. Development of our applets was done in the desktop version of GeoGebra, and then uploaded to GeoGebraTube for deployment to students.

GeoGebra applets embedded in a web site or on GeoGebraTube can be run in two modes: HTML5 mode, which requires no additional software or browser plugins, but will not work on very old web browsers, or Java mode, which requires that Java software be installed on the user's computer but can work a wider range of browsers, providing that the Java plugin is installed and appropriately configured. By default, the HTML5 version is used by GeoGebraTube but it is possible to specify that the Java version should be used instead.

3.1 Low technological barriers

A significant strength of GeoGebra, from our perspective, is its low technological barriers to use. Once uploaded to the web as HTML5, a GeoGebra applet usually runs entirely within a web browser, without requiring any additional software, plugins or configuration. This is in contrast to Java applets, which require the Java software package and browser plugin to be installed, and can require the user to click through several security warnings before the applet will load. To use a Java applet on their own PC, a student must first download and install Java, and keep it up-to-date with updates if it is to continue to work reliably. Due to the wide variety of devices and configurations, and time commitment, teaching staff typically cannot provide technical support with this process so students are 'on their own' with installing and configuring the software. We have found that, even on University-maintained PCs in computer labs, students must complete additional configuration steps (such as clicking through a security warning) before a Java applet will load. Moreover, guidance about this process must be provided by teaching staff if applet use is a required part of the course. A major strength of GeoGebra is that it removes these potential technological barriers from students; to access a GeoGebra applet, a student typically need only follow a web link, after which the applet will load and execute automatically a few seconds later. Moreover, being based on HTML5 standards, the applets 'just work' on a wide range of devices (eg. desktop PC, mobile tablet) and browsers (eg, Firefox, Safari). This greatly reduces the technological issues experienced by students and staff, and the effort expended by teaching staff providing technical support.

3.2 Ease of development

From a developer's perspective, we found building applets in GeoGebra to be very fast. Similarly to other dynamic geometry systems, GeoGebra operates by construction: each mathematical object or visual element is constructed by specifying its relationship with previously defined objects. If an object is changed, for instance as a result of a user interaction, then all objects descended from it are automatically updated to reflect the change. Hence the work required to construct a mathematical

applet using GeoGebra is close to the theoretical minimum: to fully specify an applet, the developer must, at some point, define each object in the applet and their relationship to other objects. Building 'by construction' in GeoGebra means that these definitions are, in many cases, all that is required; once an object is defined, GeoGebra takes care of rendering, updates and user interaction with the object automatically.

To quantify the development time required for such applets, we can look at the time spent by research assistants developing applets for the project. Of the applets produced by the project, 10 were developed primarily by casual research assistants employed by the project (8 of which are fully complete, 1 has minor modifications pending, and 1 has substantial development work remaining). The research assistants had no prior experience working with GeoGebra before taking on this work, and were required to learn the system as they went. The 10 applets produced by research assistants, and the time taken by the research assistants, are given in Table 3. The table also gives an indication of the technical complexity of the applet. The technical complexity is mainly determined by the number of elements in the construction and the presence or absence of multi-stage animations. The average development time required for these applets was 18.2 hours. The development work for each applet was typically spread over 2-4 weeks. The times given in Table 3 are the times spent implementing the applet in GeoGebra; they do not include time spent in the design or feedback phases (which in some cases drew on hands-on experience with students going back years), but they do include time spent implementing changes as a result of feedback.

Applet	Technical complexity	Research assistant development time (hours)	Team member development time estimate (hours)	Total development time (hours)
Partitioning of variability in ANOVA	High	25	4	29
Partitioning of variability in regression	High	18	4	22
Maximum likelihood estimators	Low	16	1	17
Distribution of order statistics	Low	10	1	11
Discrete and continuous distributions	Low	9	1	10
Prisoner's Paradox	Medium	15	2	17
QQ plots	Medium	17	2	19
Spring with forced vibrations	High	18	4	22
Central Limit Theorem and Law of Large Numbers	Low	3	1*	4
Random variables	Very high	25	6*	31
	Average	15.6	2.6	18.2

Table 3: Development times for applets developed by research assistants.

*At time of writing these applets are still under development so these times are forecasts.

For these applets, initial development was done by a research assistant under guidance from the project team. During the feedback cycle, minor changes or corrections were usually made directly by the team members. Team members did not formally record the hours spent on these tasks so the times given for team members are estimates. In some cases, technical issues arose during the development of low or medium complexity applets (as discussed in sections 3.4 and 3.5) which increased the development time required. In other cases, changes were made to the design while development was underway; the consequent modifications also increased development time.

3.3 Community support

The documentation provided by the GeoGebra project is generally reliable and comprehensive. Moreover, the large pool of resources that are available on GeoGebraTube or elsewhere on the web provide many sources of ideas or techniques to draw from. Finally, the forums are a good source of expertise or help for technical issues; typically, our posts would be answered within a day with useful suggestions for resolving problems.

3.4 Poor performance

The main obstacle that we encountered with GeoGebra as a platform for applet development was poor responsiveness of certain applets when running in HTML mode. In some cases, animations would be jerky or a noticeable lag would occur between a user interaction (for instance, clicking a button) and the response appearing on screen. This was a problem mainly for larger applets with many components, extensive computational requirements, or complex animations, and generally only when the applet was displayed in HTML mode. Performance varied with hardware and software configuration. Sometimes, performance could be improved significantly by redesigning aspects of the construction to be more efficient. In one case, where the performance in HTML mode was unacceptable, we elected to use the Java version of the applet by default, as the Java version generally has better performance, but at the cost of potentially higher technical barriers such as Java security warnings, update notifications or plugin problems.

3.5 Immaturity of software

Over the course of our project we ran into several bugs or other software issues with GeoGebra, for instance cases where a function does not work as expected, or constructions which cause a crash. In a small number of cases, an applet had to be modified or redesigned to avoid bugs or technical limitations. However, the GeoGebra developers were quick to respond to reports of such issues on the forum, often providing a fix within a day or two. We also note that GeoGebra's library of built-in mathematical functions, although sufficient for most needs in introductory calculus and statistics, is not as extensive as that of more specialised mathematical software such as Mathematica or Maple, which may be a limitation for more advanced applications.

4 Further work

Integration of the new applets into the teaching activities of the school is continuing. Several of the applets are being used in teaching for the first time in semester 2, 2015, so a full picture of the impact of the project may not be seen until 2016. One particular challenge is to explore how to successfully integrate online applet use into interactive 'whiteboard' tutorial classes (12). Applets have many attributes that potentially encourage interaction, groupwork and creativity, key features of the whiteboard tutorial style, but it is not clear to us how to leverage these aspects in a way that is not disruptive to the active work taking place at the whiteboards, and does not disadvantage students who did not bring a suitable mobile device to the tutorial.

There are several avenues to investigate further in the project evaluation. Additional evaluation is required of the supporting resources (instructors' notes and online tutorials). Do the resources provide the kind of support needed by teaching staff? Evaluation of the impact of the project on both student learning and staff professional development will also continue, guided by the framework from section 2.7, which in turn may lead to further refinement of the framework.

5 Conclusion

We have described a project to develop interactive applets targeting specific learning & teaching needs in our school's undergraduate teaching. Over 20 applets were produced by the project, covering a range of subjects and concepts. An iterative, consultative approach was taken to design the applets, with repeated cycles of feedback between the project team and relevant academics. This approach led to applets which were highly tailored to our local learning & teaching context, as well as fostering collaboration and discussion between a range of staff, and prompting academics to reflect on their teaching practice. Hence the impact of the project extended beyond the immediate utility of the applets themselves for teaching & learning, by encouraging professional development and reflective practice amongst academics. Further assessment of this impact is a focus for future evaluation. Preliminary evaluation with students shows that students are generally favourable towards the applets and feel that the applets have a positive impact on learning.

The software used by the project, GeoGebra, was found to allow very rapid development of applets, without requiring programmers or other specialist skills. The resulting applets also present low technical barriers to use by students and staff. Despite occasional limitations or shortcomings with GeoGebra, overall we found it to be a very good technological platform for this kind of project.

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